

# Exchange Rate Disconnect in General Equilibrium

OLEG ITSKHOKI

itskhoki@Princeton.edu

DMITRY MUKHIN

mukhin@Princeton.edu

Kyiv School of Economics

May 2017

## Motivation

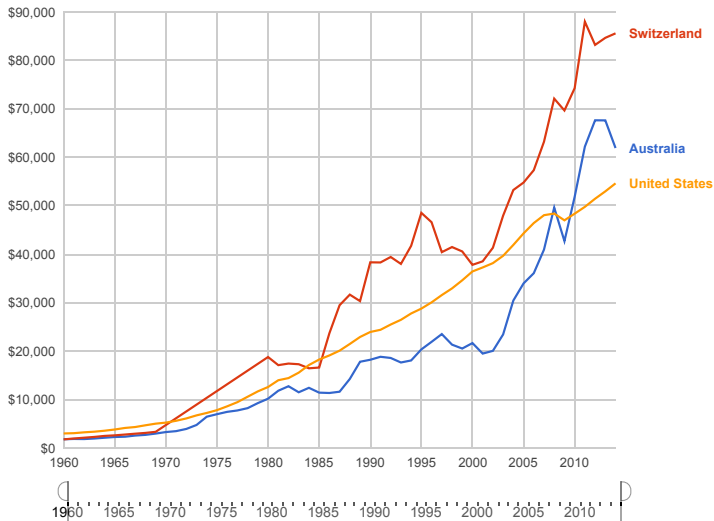
- Exchange Rate Disconnect (ERD) is one of the most pervasive and challenging puzzles in macroeconomics
  - exchange rates are present in all international macro models
  - yet, we do not have a satisfactory theory of exchange rates

## Motivation

- Exchange Rate Disconnect (ERD) is one of the most pervasive and challenging puzzles in macroeconomics
  - exchange rates are present in all international macro models
  - yet, we do not have a satisfactory theory of exchange rates
- Broader ERD combines five exchange-rate-related puzzles:
  - ① Meese-Rogoff (1983) puzzle  
*NER follows a volatile RW, uncorrelated with macro fundamentals*
  - ② PPP puzzle (Rogoff 1996)  
*RER is as volatile and persistent as NER, and the two are nearly indistinguishable at most horizons (also related [Mussa puzzle](#))*
  - ③ LOP/Terms-of-Trade puzzle (Engel 1999, Atkeson-Burstein 2008)  
*LOP violations for tradables account for nearly all RER dynamics  
ToT is three times less volatile than RER*
  - ④ Backus-Smith (1993) puzzle  
*Consumption is high when prices are high (RER appreciated)  
Consumption is five times less volatile than RER*
  - ⑤ Forward-premium puzzle (Fama 1984)  
*High interest rates predict nominal appreciations (UIP violations)*

# Motivation

GDP per capita (current US\$) ?



Data from [World Bank](#) Last updated: Jan 12, 2016

©2014 Google - [Help](#) - [Terms of Service](#) - [Privacy](#) - [Disclaimer](#) - [Discuss](#)

# Motivation

**GBP/USD (GBPUSD=X)** 1.3304 -0.0047 (-0.3499%) As of 10:16 AM EDT. CCY Delayed Price. Market open.



## Our Approach

- The literature has tried to address one puzzle at a time, often at the expense of aggravating the other puzzles
- We provide a **unifying theory of exchange rates**, capturing simultaneously all stylized facts about their properties

## Our Approach

- The literature has tried to address one puzzle at a time, often at the expense of aggravating the other puzzles
- We provide a **unifying theory of exchange rates**, capturing simultaneously all stylized facts about their properties
- A theory of exchange rate (disconnect) must specify:
  - ① The **exogenous shock process** driving the exchange rate
    - little empirical guidance here
    - we prove theoretically that only the **financial shock** is a likely candidate and then show its quantitative performance
  - ② The **transmission mechanism** muting the response of the macro variables to exchange rate movements relies on:
    - a) **strategic complementarities** in price setting resulting in PTM
    - b) **weak substitutability** between home and foreign goods
    - c) **home bias** in consumption
    - d) **monetary policy rule** stabilizing domestic inflation
    - all admitting tight empirical discipline
    - **nominal rigidities** are not essential

▶ show

## Contributions

- A dynamic general equilibrium model of exchange rate
  - fully analytically tractable, yet quantitative
- Four new mechanisms:
  - ① Equilibrium exchange rate determination and dynamics  
(*cf.* Engel and West 2005)
  - ② PPP puzzle and related puzzles  
(Rogoff '96, CKM '02, Kehoe and Midrigan '08, Monacelli '04)
  - ③ Backus-Smith puzzle  
(*cf.* Corsetti, Dedola and Leduc 2008)
  - ④ Forward premium puzzle  
(Engel 2016)



# MODELING FRAMEWORK

## Model setup

- Two countries: home (Europe) and foreign (US, denoted w/\*)
- Nominal wages  $W_t$  in euros and  $W_t^*$  in dollars, the numeraire
- $\mathcal{E}_t$  is the **nominal exchange rate** (price of one dollar in euros)
- Baseline model:
  - representative households
  - representative firms
  - one internationally-traded foreign-bond
- We allow for all possible shocks/CKM-style wedges:

$$\Omega_t = (w_t, \chi_t, \kappa_t, a_t, g_t, \mu_t, \eta_t, \xi_t, \psi_t)$$

and foreign counterparts

# Equilibrium conditions

## 1 Households:

- (i) labor supply and asset demand [▶ show](#)
- (ii) expenditure on home and foreign good [▶ show](#)
  - $\gamma$  expenditure share on foreign goods
  - $\theta$  elasticity of substitution between home and foreign goods

## 2 Firms:

- (i) production and profits [▶ show](#)
- (ii) price setting [▶ show](#)
  - $\alpha$  strategic complementarity elasticity in price setting

## 3 Government: balanced budget [▶ show](#)

## 4 Foreign: symmetric [▶ show](#)

## 5 GE: market clearing and country budget constraint [▶ show](#)

# DISCONNECT IN THE LIMIT

## Disconnect in the Autarky Limit

- Consider an economy in **autarky** = complete ER disconnect
  - (i) NER is not determined and can take any value
  - (ii) this has no effect on domestic quantities, prices or interest rates
  - (iii) as price levels are determined independently from NER, RER moves one-to-one with NER
- + the further from autarky, the less likely the disconnect [▶ show](#)

## Disconnect in the Autarky Limit

- Consider an economy in **autarky** = complete ER disconnect
  - (i) NER is not determined and can take any value
  - (ii) this has no effect on domestic quantities, prices or interest rates
  - (iii) as price levels are determined independently from NER, RER moves one-to-one with NER

+ the further from autarky, the less likely the disconnect

[▶ show](#)

- **Definition:** Exchange rate disconnect in the autarky limit

$$\lim_{\gamma \rightarrow 0} \frac{d\mathbf{Z}_{t+j}}{d\varepsilon_t} = \mathbf{0} \quad \forall j \quad \text{and} \quad \lim_{\gamma \rightarrow 0} \frac{d\mathcal{E}_t}{d\varepsilon_t} \neq 0.$$

## Disconnect in the Autarky Limit

- Consider an economy in **autarky** = complete ER disconnect
  - (i) NER is not determined and can take any value
  - (ii) this has no effect on domestic quantities, prices or interest rates
  - (iii) as price levels are determined independently from NER, RER moves one-to-one with NER

+ the further from autarky, the less likely the disconnect

[▶ show](#)

- **Definition:** Exchange rate disconnect in the autarky limit

$$\lim_{\gamma \rightarrow 0} \frac{d\mathbf{Z}_{t+j}}{d\varepsilon_t} = \mathbf{0} \quad \forall j \quad \text{and} \quad \lim_{\gamma \rightarrow 0} \frac{d\varepsilon_t}{d\varepsilon_t} \neq 0.$$

- **Proposition 1:** *The model cannot exhibit exchange rate disconnect in the limit with zero weight on:*
  - (i) LOP deviation shocks:  $\eta_t$
  - (ii) Foreign-good demand shocks:  $\xi_t$
  - (iii) Financial (international asset demand) shocks:  $\psi_t$
- A pessimistic result for IRBC and NOEM models

## Admissible Shocks

- Intuition: two international conditions

- risk sharing: 
$$\mathbb{E}_t \left\{ R_{t+1}^* \left[ \Theta_{t+1}^* - \Theta_{t+1} \frac{\varepsilon_{t+1}}{\varepsilon_t} e^{\psi_t} \right] \right\} = 0$$

- budget constraint: 
$$B_{t+1}^* - R_t^* B_t^* = NX^*(Q_t; \eta_t, \xi_t)$$

- In the limit, shocks to these conditions have a vanishingly small effect, while other shocks still have a direct effect



## Admissible Shocks

- Intuition: two international conditions

- risk sharing: 
$$\mathbb{E}_t \left\{ R_{t+1}^* \left[ \Theta_{t+1}^* - \Theta_{t+1} \frac{\varepsilon_{t+1}}{\varepsilon_t} e^{\psi_t} \right] \right\} = 0$$

- budget constraint: 
$$B_{t+1}^* - R_t^* B_t^* = NX^*(Q_t; \eta_t, \xi_t)$$

- In the limit, shocks to these conditions have a vanishingly small effect, while other shocks still have a direct effect

- **Proposition 2:** *In the autarky limit,  $\psi_t$  is the only shock that simultaneously and robustly produces:*

- (i) *positively correlated ToT and RER (Obstfeld-Rogoff moment)*

- (ii) *negatively correlated relative consumption growth and real exchange rate depreciations (Backus-Smith correlation)*

- (iii) *deviations from the UIP (negative Fama coefficient).*

⇒  $\psi_t$  is the prime candidate shock for a **quantitative** model of ER disconnect

# **BASELINE MODEL**

OF EXCHANGE RATE DISCONNECT

## Ingredients

- ① Financial exchange rate shock  $\psi_t$  only:

▶ microfoundations

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t$$

- persistent ( $\rho \lesssim 1$ , e.g.  $\rho = 0.97$ ) w/small innovations ( $\sigma_\varepsilon \gtrsim 0$ ):

$$\psi_t = \rho\psi_{t-1} + \varepsilon_t, \quad \beta\rho < 1$$

- important limiting case:  $\beta\rho \rightarrow 1$

# Ingredients

## 1 Financial exchange rate shock $\psi_t$ only:

▶ microfoundations

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t$$

— persistent ( $\rho \lesssim 1$ , e.g.  $\rho = 0.97$ ) w/small innovations ( $\sigma_\varepsilon \gtrsim 0$ ):

$$\psi_t = \rho \psi_{t-1} + \varepsilon_t, \quad \beta \rho < 1$$

— important limiting case:  $\beta \rho \rightarrow 1$

## 2 Transmission mechanism

(i) Strategic complementarities:  $\alpha = 0.4$  (AIK 2015)

(ii) Elasticity of substitution:  $\theta = 1.5$  (FLOR 2014)

(iii) Home bias:  $\gamma = 0.07 = \frac{1}{2} \frac{\text{Imp} + \text{Exp}}{\text{GDP}} \frac{\text{GDP}}{\text{Prod-n}}$  (for US, EU, Japan)

• Monetary regime:  $W_t \equiv 1$  and  $W_t^* \equiv 1$

• Other parameters:

$$\beta = 0.99, \quad \sigma = 2, \quad \nu = 1, \quad \phi = 0.5, \quad \zeta = 1 - \phi$$

# Roadmap

- ① Equilibrium exchange rate dynamics
- ② Real and nominal exchange rates
- ③ Exchange rate and prices
- ④ Exchange rate and quantities
- ⑤ Exchange rate and interest rates

## Exchange Rate Dynamics

- ① The international risk sharing condition:

$$\underbrace{i_t - i_t^*}_{\propto \gamma \psi_t} - \mathbb{E}_t \Delta e_{t+1} = \psi_t \quad \Rightarrow \quad \mathbb{E}_t \Delta e_{t+1} = -\frac{1}{1 + \gamma \lambda_1} \psi_t$$

- ② Intertemporal budget constraint:

$$\beta b_{t+1}^* - b_t^* = n x_t, \quad n x_t = \gamma \lambda_2 \cdot e_t$$

## Exchange Rate Dynamics

- ① The international risk sharing condition:

$$\underbrace{i_t - i_t^*}_{\propto \gamma \psi_t} - \mathbb{E}_t \Delta e_{t+1} = \psi_t \quad \Rightarrow \quad \mathbb{E}_t \Delta e_{t+1} = -\frac{1}{1 + \gamma \lambda_1} \psi_t$$

- ② Intertemporal budget constraint:

$$\beta b_{t+1}^* - b_t^* = n x_t, \quad n x_t = \gamma \lambda_2 \cdot e_t$$

### Proposition

When  $\psi_t \sim AR(1)$ , the equilibrium exchange rate follows ARIMA:

$$\Delta e_t = \rho \Delta e_{t-1} + \frac{1}{1 + \gamma \lambda_1} \frac{\beta}{1 - \beta \rho} \left( \varepsilon_t - \frac{1}{\beta} \varepsilon_{t-1} \right).$$

This process becomes arbitrary close to a random walk as  $\beta \rho \rightarrow 1$ .

— This is the unique equilibrium solution, bubble solutions do not exist

## Exchange Rate Dynamics

- ① The international risk sharing condition:

$$\underbrace{i_t - i_t^*}_{\propto \gamma \psi_t} - \mathbb{E}_t \Delta e_{t+1} = \psi_t \quad \Rightarrow \quad \mathbb{E}_t \Delta e_{t+1} = -\frac{1}{1 + \gamma \lambda_1} \psi_t$$

- ② Intertemporal budget constraint:

$$\beta b_{t+1}^* - b_t^* = n x_t, \quad n x_t = \gamma \lambda_2 \cdot e_t$$

### Proposition

When  $\psi_t \sim AR(1)$ , the equilibrium exchange rate follows ARIMA:

$$(1 - \rho L) \Delta e_t = \frac{1}{1 + \gamma \lambda_1} \frac{\beta}{1 - \beta \rho} \left( 1 - \frac{1}{\beta} L \right) \varepsilon_t.$$

This process becomes arbitrary close to a random walk as  $\beta \rho \rightarrow 1$ .

— This is the unique equilibrium solution, bubble solutions do not exist



# Exchange Rate Dynamics

- ① The international risk sharing condition:

$$\underbrace{i_t - i_t^*}_{\propto \gamma \psi_t} - \mathbb{E}_t \Delta e_{t+1} = \psi_t \quad \Rightarrow \quad \mathbb{E}_t \Delta e_{t+1} = -\frac{1}{1 + \gamma \lambda_1} \psi_t$$

- ② Intertemporal budget constraint:

$$\beta b_{t+1}^* - b_t^* = n x_t, \quad n x_t = \gamma \lambda_2 \cdot e_t$$

## Proposition

When  $\psi_t \sim AR(1)$ , the equilibrium exchange rate follows ARIMA:

$$(1 - \rho L) \Delta e_t = \frac{1}{1 + \gamma \lambda_1} \frac{\beta}{1 - \beta \rho} \left( 1 - \frac{1}{\beta} L \right) \varepsilon_t.$$

This process becomes arbitrary close to a random walk as  $\beta \rho \rightarrow 1$ .

- This is the unique equilibrium solution, bubble solutions do not exist
- NFA  $\Delta b_{t+1}^* \sim AR(1)$ :  $\Delta b_{t+1}^* = \frac{\gamma \lambda_2}{1 + \gamma \lambda_1} \frac{1}{1 - \beta \rho} \psi_t$

# Properties of the Exchange Rate

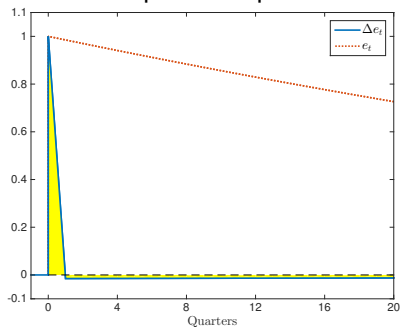
- Near-random-walk behavior (as  $\beta\rho \rightarrow 1$ ):

①  $\text{corr}(\Delta e_{t+1}, \Delta e_t) \rightarrow 0$

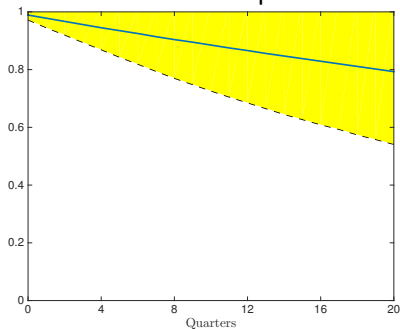
②  $\frac{\text{var}(\Delta_k e_{t+k} - \mathbb{E}_t \Delta_k e_{t+k})}{\text{var}(\Delta_k e_{t+k})} \rightarrow 1$

③  $\frac{\text{std}(\Delta e_t)}{\text{std}(\psi_t)} \rightarrow \infty$

## Impulse Response



## Variance Decomposition



# PPP Puzzle

## Proposition

*RER and NER are tied together by the following relationship:*

$$q_t = \frac{1}{1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma}} e_t.$$

- $(q_t - e_t) \xrightarrow{\gamma \rightarrow 0} 0$
- Relative volatility:  $\frac{\text{std}(\Delta q_t)}{\text{std}(\Delta e_t)} = \frac{1}{1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma}} = 0.75$
- Heterogenous firms and/or LCP sticky prices further increase volatility of RER

# PPP Puzzle

## Intuition

- Real exchange rate:

$$Q = \frac{P_t^* \mathcal{E}_t}{P_t}$$

- ① either  $P_t$  and  $P_t^*$  are very sticky (+ monetary shocks); or
- ② or economies are very closed,  $\gamma \approx 0$  (+  $\psi_t$  shocks)

# PPP Puzzle

## Intuition

- Real exchange rate:

$$Q = \frac{P_t^* \mathcal{E}_t}{P_t}$$

- ① either  $P_t$  and  $P_t^*$  are very sticky (+ monetary shocks); or
  - ② or economies are very closed,  $\gamma \approx 0$  (+  $\psi_t$  shocks)
- Intuition (failure of IRBC and NOEM models):

$$p_t = (w_t - a_t) + \frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} q_t$$

$$p_t^* = (w_t^* - a_t^*) - \frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} q_t$$

$$\Rightarrow \left[ 1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma} \right] q_t = e_t + (w_t^* - a_t^*) - (w_t - a_t)$$

## Exchange Rates and Prices

- Three closely related variables:

$$Q_t = \frac{P_t^* \mathcal{E}_t}{P_t} \quad Q_t^P = \frac{P_{Ft}^* \mathcal{E}_t}{P_{Ht}} \quad S_t = \frac{P_{Ft}}{P_{Ht}^* \mathcal{E}_t}$$

- Two relationships:

$$q_t = (1 - \gamma)q_t^P - \gamma s_t$$
$$s_t = q_t^P - 2\alpha q_t$$

- In the data:  $q_t^P \approx q_t$ ,  $\text{std}(\Delta q_t) \gg \text{std}(\Delta s_t)$ ,  $\text{corr}(\Delta s_t, \Delta q_t) > 0$

- **Proposition:**

$$q_t^P = \frac{1 - 2\alpha\gamma}{1 - 2\gamma} q_t \quad \text{and} \quad s_t = \frac{1 - 2\alpha(1 - \gamma)}{1 - 2\gamma} q_t$$

- conventional models with  $\alpha = 0$  cannot do the trick
- $\alpha$  needs to be positive, but not too large

## Exchange Rates and Prices

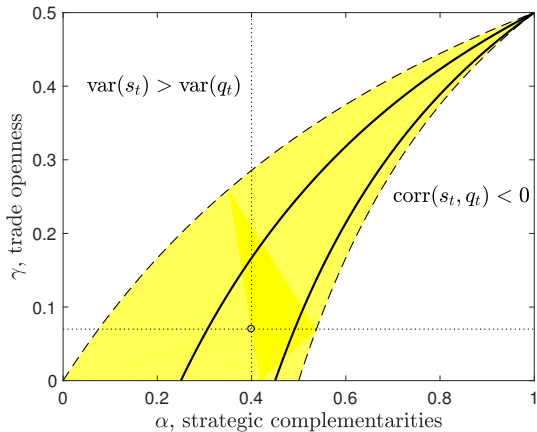


Figure: Terms of trade and Real exchange rate

# Exchange Rate and Quantities

## Backus-Smith puzzle

- Static relationship between consumption and RER: ▶ show
  - (i) labor supply
  - (ii) labor demand
$$\left. \begin{array}{l} \text{(i) labor supply} \\ \text{(ii) labor demand} \end{array} \right\} \sigma \tilde{c}_t + \frac{1}{\nu} \tilde{y}_t = -\gamma q_t$$
- (iii) goods market clearing:  $\tilde{y}_t = (1 - 2\gamma)\tilde{c}_t + 2\theta(1 - \alpha)\gamma q_t$
- “Dismiss” asset market (Backus-Smith) condition:  
$$\sigma \tilde{c}_t = q_t \quad \text{vs.} \quad \mathbb{E}_t\{\Delta \tilde{c}_{t+1} - \Delta q_{t+1}\} = \psi_t$$



# Exchange Rate and Quantities

## Backus-Smith puzzle

- Static relationship between consumption and RER: [▶ show](#)
  - (i) labor supply
  - (ii) labor demand
  - (iii) goods market clearing:
$$\left. \begin{array}{l} \text{(i) labor supply} \\ \text{(ii) labor demand} \end{array} \right\} \sigma \tilde{c}_t + \frac{1}{\nu} \tilde{y}_t = -\gamma q_t$$
$$\text{(iii) goods market clearing: } \tilde{y}_t = (1 - 2\gamma)\tilde{c}_t + 2\theta(1 - \alpha)\gamma q_t$$
- “Dismiss” asset market (Backus-Smith) condition:
$$\sigma \tilde{c}_t = q_t \quad \text{vs.} \quad \mathbb{E}_t\{\Delta \tilde{c}_{t+1} - \Delta q_{t+1}\} = \psi_t$$

- **Proposition:** Static expenditure switching implies:

$$c_t - c_t^* = -\frac{2\theta(1 - \alpha)(1 - \gamma) + \nu}{(1 - 2\gamma) + \sigma\nu} \frac{2\gamma}{1 - 2\gamma} q_t$$

# Exchange Rate and Quantities

## Backus-Smith puzzle

- Static relationship between consumption and RER: ▶ show
    - (i) labor supply
    - (ii) labor demand } 
$$\sigma \tilde{c}_t + \frac{1}{\nu} \tilde{y}_t = -\gamma q_t$$
  - (iii) goods market clearing: 
$$\tilde{y}_t = (1 - 2\gamma)\tilde{c}_t + 2\theta(1 - \alpha)\gamma q_t$$
- “Dismiss” asset market (Backus-Smith) condition:  
$$\sigma \tilde{c}_t = q_t \quad \text{vs.} \quad \mathbb{E}_t\{\Delta \tilde{c}_{t+1} - \Delta q_{t+1}\} = \psi_t$$

- **Proposition:** Static expenditure switching implies:

$$c_t - c_t^* = -\frac{2\theta(1 - \alpha)(1 - \gamma) + \nu}{(1 - 2\gamma) + \sigma\nu} \frac{2\gamma}{1 - 2\gamma} q_t + \kappa(a_t - a_t^*)$$

- Three alternatives in the literature to get BS puzzle:
  - 1 Super-persistent (news-like) shocks (CC 2013)
  - 2 Low elasticity of substitution  $\theta < 1$  (CDL 2008)
  - 3 Non-tradable productivity shocks (BT 2008)

## Exchange Rate and Quantities

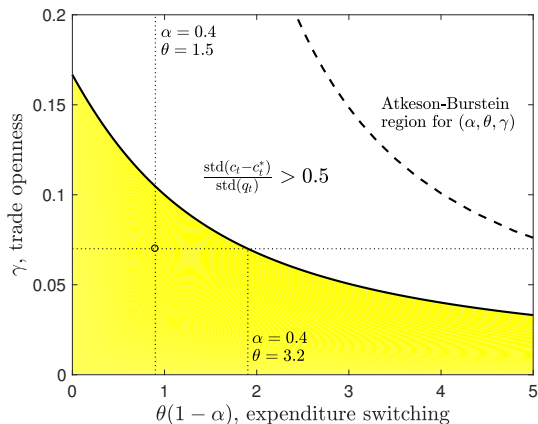


Figure: Exchange rate disconnect: relative consumption volatility

## Exchange Rate and Interest rates

- Two equilibrium conditions:

$$\psi_t = (i_t - i_t^*) - \mathbb{E}_t \Delta e_{t+1} \quad \text{and} \quad i_t - i_t^* = -\gamma \lambda_1 \mathbb{E}_t \Delta e_{t+1}$$

### Proposition

*Fama-regression coefficient:*

$$\mathbb{E}\{\Delta e_{t+1} | i_{t+1} - i_{t+1}^*\} = \beta_F (i_{t+1} - i_{t+1}^*), \quad \beta_F \equiv -\frac{1}{\gamma \lambda_1} < 0.$$

*In the limit  $\beta \rho \rightarrow 1$ :*

- (i) *Fama-regression  $R^2 \rightarrow 0$*
- (ii)  $\text{var}(i_t - i_t^*) / \text{var}(\Delta e_{t+1}) \rightarrow 0$
- (iii)  $\rho(\Delta e_t) \rightarrow 0$ , while  $\rho(i_t - i_t^*) \rightarrow 1$
- (iv) *the Sharpe ratio of the carry trade:  $SR_C \rightarrow 0$*

*\* carry trade return:  $r_{t+1}^C = x_t \cdot (i_t - i_t^* - \Delta e_{t+1})$  with  $x_t = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$*

# EXTENSIONS

## Extensions

- 1 Monetary model with nominal rigidities and a Taylor rule
  - different transmission mechanism
  - similar quantitative conclusions for  $\psi_t$  shock
  - Mussa puzzle
- 2 Multiple shocks:
  - productivity, monetary, foreign good and asset demand
  - variance decomposition: contribution of  $\psi_t \approx 70\%$
- 3 Financial model with noise traders and limits to arbitrage (De Long et al 1990, Jeanne and Rose 2002)
  - A model of upward slopping supply in asset markets with endogenous equilibrium volatility of  $\psi_t$  and  $\Delta e_{t+1}$
  - Stationary model with similar small sample properties
  - Additional moments: the Engel (2016) “risk premium” puzzle
- 4 Robustness to parameters [▶ show](#)

## Monetary model

- Standard New Keynesian Open Economy model
- Baseline: sticky wages and LCP sticky prices
- Taylor rule:  $i_t = \rho_i i_{t-1} + (1 - \rho_i) \delta_\pi \pi_t + \varepsilon_t^m$
- New transmission:  $i_t$  does not respond directly to the  $\psi_t$  shock, but instead through inflation it generates
- Results:
  - ① monetary shock alone results in numerous ER puzzles
  - ② financial shock  $\psi_t$  has quantitative similar properties, with two exceptions:
    - + makes RER more volatile and NER closer to a random walk
    - RER is negatively correlated with ToT (see Gopinath et al)

# Model comparison

Moment	Data	A: SINGLE-SHOCK MODELS				B: MULTI-SHOCK MODELS		
		Fin. shock $\psi$		NOEM	IRBC	NOEM	IRBC	Financial
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\rho(\Delta e)$	0.00	-0.02 (0.09)	-0.03 (0.09)	-0.05 (0.09)	0.00 (0.09)	-0.03 (0.09)	-0.02 (0.09)	-0.01 (0.09)
$\rho(q)$	0.95	0.93 (0.04)	0.91 (0.05)	0.84 (0.05)	0.93 (0.04)	0.93 (0.04)	0.93 (0.04)	0.93 (0.04)
$\sigma(\Delta q)/\sigma(\Delta e)$	0.99	0.79	0.97	0.97	1.64	0.98	0.94	0.76
$\text{corr}(\Delta q, \Delta e)$	0.98	1	1	0.99	0.99	1.00	0.97	0.94
$\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q)$	0.20	0.31	0.12	0.52	0.64	0.20	0.30	0.31
$\text{corr}(\Delta c - \Delta c^*, \Delta q)$	-0.20	-1	-0.95	1	1	-0.20 (0.09)	-0.20 (0.09)	-0.22 (0.09)
$\sigma(\Delta nx)/\sigma(\Delta q)$	0.10	0.26	0.17	0.08	0.14	0.32	0.30	0.10
$\text{corr}(\Delta nx, \Delta q)$	$\approx 0$	1	0.99	1	1	-0.00 (0.09)	-0.00 (0.09)	-0.02 (0.09)
$\sigma(\Delta s)/\sigma(\Delta e)$	0.35	0.23	0.80	0.82	0.49	0.80	0.28	0.23
$\text{corr}(\Delta s, \Delta e)$	0.60	1	-0.93	-0.96	0.99	-0.93	0.97	0.94
Fama $\beta$	$\lesssim 0$	-2.4 (1.7)	-3.4 (2.6)	1.2 (0.7)	1.4 (0.5)	-0.6 (1.4)	-0.7 (1.3)	-2.8 (3.5)
Fama $R^2$	0.02	0.03 (0.02)	0.03 (0.02)	0.03 (0.03)	0.09 (0.02)	0.00 (0.01)	0.00 (0.01)	0.01 (0.02)
$\sigma(i - i^*)/\sigma(\Delta e)$	0.06	0.07 (0.02)	0.05 (0.02)	0.14 (0.02)	0.21 (0.06)	0.06 (0.02)	0.08 (0.02)	0.03 (0.01)
$\rho(i - i^*)$	0.90	0.93 (0.04)	0.98 (0.01)	0.84 (0.05)	0.93 (0.04)	0.91 (0.04)	0.93 (0.04)	0.90 (0.04)
Carry $SR$	0.20	0.21 (0.04)	0.20 (0.04)	0	0	0.17 (0.06)	0.19 (0.06)	0.12 (0.07)



## Variance decomposition

Shocks		NOEM		IRBC	
		$\text{var}(\Delta e_t)$	$\text{var}(\Delta q_t)$	$\text{var}(\Delta e_t)$	$\text{var}(\Delta q_t)$
Monetary (Taylor rule)	$\varepsilon_t^m$	10%	10%	—	—
Productivity	$a_t$	—	—	3%	9%
Foreign-good demand	$\xi_t$	19%	20%	23%	39%
Financial	$\psi_t$	71%	70%	74%	52%

## Mussa puzzle

Moment	Data	Model	
		(1)	(2)
$\text{std}(\Delta e_t)$	0.13	0.13	0.13
$\text{std}(\Delta q_t)$	0.26	0.18	0.16
$\text{corr}(\Delta q_t, \Delta e_t)$	0.66	0.79	0.84
$\text{std}(\Delta c_t - \Delta c_t^*)$	$\approx 1$	2.63	1.33
$\text{corr}(\Delta c_t - \Delta c_t^*, \Delta q_t)$	$> 0$	-0.63	0.13
Fama $\beta$	$> 0$	-0.1	1.1

## Financial model

- Symmetric countries with international bond holding intermediated by a financial sector
- Three type of agents:  $B_{t+1}^* + N_{t+1}^* + D_{t+1}^* = 0$
- Noise traders:  $N_{t+1}^* = n (e^{\psi_t} - 1)$
- Arbitrageurs:  $\max_d \left\{ d \mathbb{E}_t \tilde{R}_{t+1} - \frac{\omega}{2} \text{var}_t(\tilde{R}_{t+1}) d^2 \right\}$ ,  $\tilde{R}_{t+1}^* \equiv R_t^* - R_t \frac{\varepsilon_t}{\varepsilon_{t+1}}$   
results in bond supply:

$$D_{t+1}^* = m \frac{\mathbb{E}_t \tilde{R}_{t+1}}{\omega \text{var}_t(\tilde{R}_{t+1})}$$

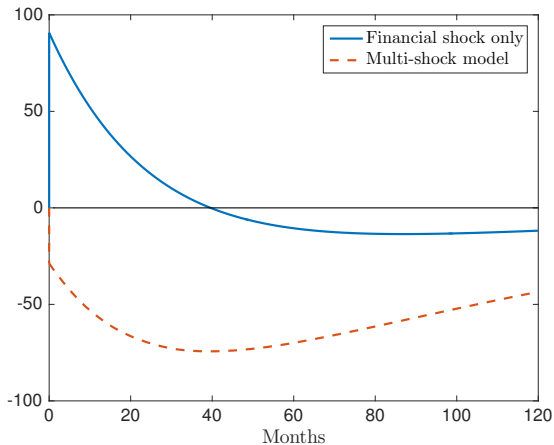
- Generalized UIP condition:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}, \quad \chi_1 \equiv \frac{n/\beta}{m/(\omega \sigma_e^2)}, \quad \chi_2 \equiv \frac{\bar{Y}}{m/(\omega \sigma_e^2)}$$

- **Proposition:**  $e_t$  and  $q_t$  follow an ARMA(2,1), but with the same near-random-walk properties.

## Engel (2016) “risk premium” puzzle

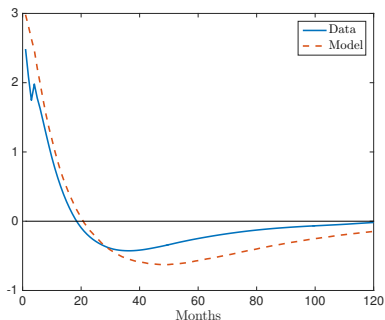
Figure: Response of  $e_{t+j}$  to innovation in  $i_t - i_t^*$



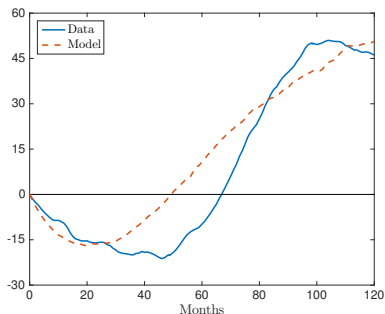
# Engel (2016) “risk premium” puzzle

Figure: Projections on  $i_t - i_t^*$

(a) Risk premium,  $\mathbb{E}_t \rho_{t+j}$



(b) Exchange rate,  $e_{t+j} - e_t$



where  $\rho_t = i_t - i_t^* - \Delta e_{t+1}$

## Conclusion

- Exchange rates have been very puzzling for macroeconomists
- We offer a unifying quantitative GE theory of exchange rates
- Which international macro results are robust?
  - Monetary policy transmission and spillovers: likely yes
  - Welfare analysis and optimal exchange rate regimes: likely no
- Our tractable macro GE environment can be useful for both:
  - ① empirical/quantitative studies of ER and transmission
  - ② financial models of exchange rates

# APPENDIX

# Puzzle Resolution Mechanism

◀ back to slides

Puzzle		Ingredients	
Meese-Rogoff, UIP	→ {	<ul style="list-style-type: none"><li>• persistent financial shock</li><li>• conventional Taylor rule</li></ul>	$\psi_t$
PPP	+	<ul style="list-style-type: none"><li>• home bias</li></ul>	$\gamma$
Terms-of-trade	+	<ul style="list-style-type: none"><li>• strategic complementarities</li></ul>	$\alpha$
Backus-Smith	+	<ul style="list-style-type: none"><li>• weak substitutability</li></ul>	$\theta$



# Puzzle Resolution Mechanism

◀ back to slides

Puzzle		Ingredients	
Meese-Rogoff, UIP	→ {	• persistent financial shock • conventional Taylor rule	$\psi_t$
PPP	+	• home bias	$\gamma$
Terms-of-trade	+	• strategic complementarities	$\alpha$
Backus-Smith	+	• weak substitutability	$\theta$

- Parameter restrictions:
  - 1 Marshall-Lerner condition:  $\theta > 1/2$
  - 2 Nominal UIP:  $\theta > IES$

## New Mechanisms

- ① Exchange rate dynamics:
  - near random-walk behavior emerging from the intertemporal budget constraint under incomplete markets
  - small but persistent expected appreciations require a large unexpected devaluation on impact
- ② PPP puzzle
  - no wedge between nominal and real exchange rates, unlike IRBC and NOEM models
- ③ Violation of the Backus-Smith condition:
  - we demote the dynamic risk-sharing condition from determining consumption allocation
  - instead static market clearing determination of consumption
- ④ Violation of UIP and Forward premium puzzle:
  - small persistent interest rate movements support consumption allocation, disconnected from volatile exchange rate
  - negative Fama coefficient, yet small Sharpe ratio on carry trade

## Households

◀ back to slides

- Representative home household solves:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\chi_t} \left( \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{e^{\kappa_t}}{1+1/\nu} L_t^{1+1/\nu} \right)$$
$$\text{s.t. } P_t C_t + \frac{B_{t+1}}{R_t} + \frac{B_{t+1}^* \mathcal{E}_t}{e^{\psi_t} R_t^*} \leq B_t + B_t^* \mathcal{E}_t + W_t L_t + \Pi_t + T_t$$

- Household optimality (labor supply and demand for bonds):

$$e^{\kappa_t} C_t^\sigma L_t^{1/\nu} = \frac{W_t}{P_t},$$
$$R_t \mathbb{E}_t \{ \Theta_{t+1} \} = 1,$$
$$e^{\psi_t} R_t^* \mathbb{E}_t \left\{ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \Theta_{t+1} \right\} = 1,$$

where the home nominal SDF is given by:

$$\Theta_{t+1} \equiv \beta e^{\Delta \chi_{t+1}} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}$$

- Consumption expenditure on home and foreign goods:

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft}$$

arises from a homothetic consumption aggregator:

$$C_{Ht} = (1 - \gamma) e^{-\gamma \xi_t} h\left(\frac{P_{Ht}}{P_t}\right) C_t,$$

$$C_{Ft} = \gamma e^{(1-\gamma)\xi_t} h\left(\frac{P_{Ft}}{P_t}\right) C_t$$

- The **foreign share** and the **elasticity of substitution**:

$$\gamma_t \equiv \frac{P_{Ft} C_{Ft}}{P_t C_t} \Big|_{\substack{P_{Ht}=P_{Ft}=P_t \\ \xi_t=0}} = \gamma$$

$$\theta_t \equiv - \frac{\partial \log h(x_t)}{\partial \log x_t} \Big|_{x_t=1} = \theta$$

# Production and profits

◀ back to slides

- Production function with intermediates:

$$Y_t = e^{a_t} L_t^{1-\phi} X_t^\phi$$

$$MC_t = e^{-a_t} \left( \frac{W_t}{1-\phi} \right)^{1-\phi} \left( \frac{P_t}{\phi} \right)^\phi$$

- Profits:

$$\Pi_t = (P_{Ht} - MC_t) Y_{Ht} + (P_{Ht}^* \mathcal{E}_t - MC_t) Y_{Ht}^*$$

where  $Y_t = Y_{Ht} + Y_{Ht}^*$

- Labor and intermediate goods demand:

$$W_t L_t = (1 - \phi) MC_t Y_t$$

$$P_t X_t = \phi MC_t Y_t$$

and fraction  $\gamma_t$  of  $P_t X_t$  is allocated to foreign intermediates

- We postulate the following price setting rule:

$$P_{Ht} = e^{\mu_t} MC_t^{1-\alpha} P_t^\alpha$$

$$P_{Ht}^* = e^{\mu_t + \eta_t} (MC_t / \mathcal{E}_t)^{1-\alpha} P_t^{*\alpha}$$

- LOP violations:

$$Q_{Ht} \equiv \frac{P_{Ht}^* \mathcal{E}_t}{P_{Ht}} = e^{\eta_t} Q_t^\alpha$$

where the **real exchange rate** is given by:

$$Q_t \equiv \frac{P_t^* \mathcal{E}_t}{P_t}$$

- Government runs a balanced budget, using lump-sum taxes to finance expenditure:

$$P_t G_t = P_t e^{g_t},$$

where fraction  $\gamma_t$  of  $P_t G_t$  is allocated to foreign goods

- The transfers to the households are given by:

$$T_t = (e^{-\psi_t} - 1) \frac{B_{t+1}^* \mathcal{E}_t}{R_t^*} - P_t e^{g_t}$$

- Foreign households and firms are symmetric, subject to:

$$\{\chi_t^*, \kappa_t^*, \xi_t^*, a_t^*, \mu_t^*, \eta_t^*, g_t^*\}$$

- Foreign households only differ in that they do not have access to the home bond, which is not internationally traded.

As a result, their only Euler equation is for foreign bonds:

$$R_t^* \mathbb{E}_t \{ \Theta_{t+1}^* \} = 1, \quad \Theta_{t+1}^* \equiv \beta e^{\Delta \chi_{t+1}^*} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*}$$



1 Labor market clearing

2 Goods market clearing, e.g.:

$$Y_{Ht}^* = \gamma e^{(1-\gamma)\xi_t^*} h\left(\frac{P_{Ht}^*}{P_t^*}\right) [C_t^* + X_t^* + G_t^*]$$

3 Bond market clearing:

$$B_t = 0 \quad \text{and} \quad B_t^* + B_t^{*F} = 0$$

4 Country budget constraint:

$$\frac{B_{t+1}^* \mathcal{E}_t}{R_t^*} - B_t^* \mathcal{E}_t = NX_t, \quad NX_t = P_{Ht}^* \mathcal{E}_t Y_{Ht}^* - P_{Ft} Y_{Ft},$$

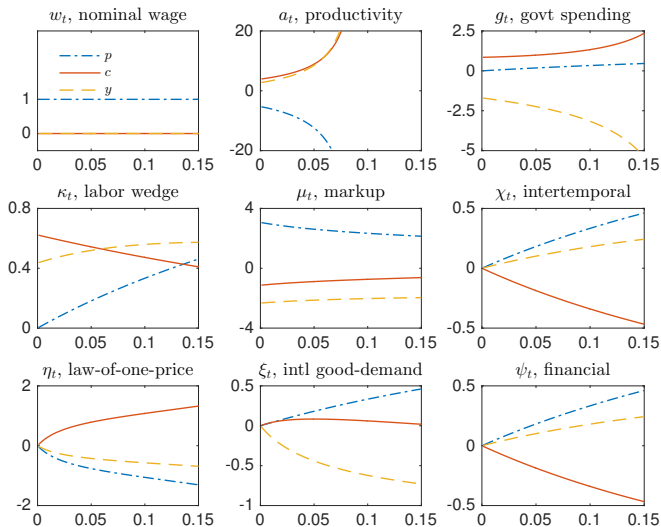
and we define the **terms of trade**:

$$S_t \equiv \frac{P_{Ft}}{P_{Ht}^* \mathcal{E}_t}$$

# Impulse responses

[◀ back](#)

The figure plots  $\frac{\partial z_t / \partial \varepsilon_t}{\partial e_t / \partial \varepsilon_t}$  for different values of  $\gamma$ , where  $z \in \{p, c, y\}$  are different macro variables and  $\varepsilon \in \Omega$  are different shocks



# Microfoundations for $\psi_t$ shock

◀ back to slides

Risk premium shock:  $\psi_t = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$

- 1 International asset demand shocks (in the utility function)  
— e.g., Dekle, Jeong and Kiyotaki (2014)
- 2 Noise trader shocks and limits to arbitrage  
— e.g., Jeanne and Rose (2002)
  - noise traders can be liquidity/safety traders
  - arbitrageurs with downward sloping demand
  - multiple equilibria → Mussa puzzle
- 3 Heterogenous beliefs or expectation shocks  
— e.g., Bacchetta and van Wincoop (2006)
  - huge volumes of currency trades (also order flows)
  - $\psi_t$  are disagreement or expectation shocks
- 4 Financial frictions (e.g., Gabaix and Maggiori 2015)
- 5 Risk premia models  
(rare disasters, long-run risk, habits, segmented markets)

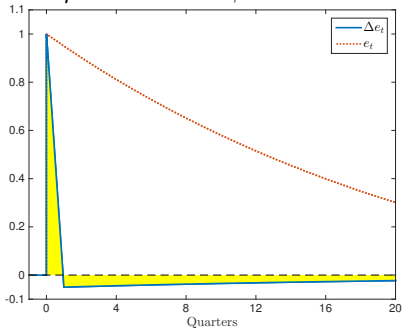
# Properties of the Exchange Rate

◀ back to slides

- Near-random-walk behavior (as  $\beta\rho \rightarrow 1$ )

$$\text{corr}(\Delta e_{t+1}, \Delta e_t) \rightarrow 0 \quad \frac{\text{var}(\Delta_k e_{t+k} - \mathbb{E}_t \Delta_k e_{t+k})}{\text{var}(\Delta_k e_{t+k})} \rightarrow 1 \quad \frac{\text{std}(\Delta e_t)}{\text{std}(\psi_t)} \rightarrow \infty$$

$\rho = 0.96$  and  $\beta = 0.99$



$\rho = 0.99$  and  $\beta = 0.995$

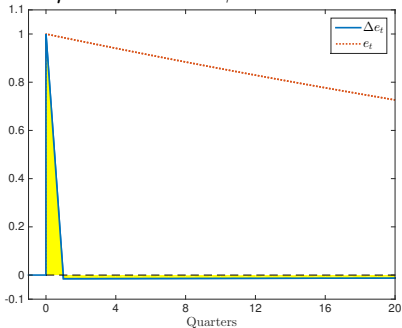


Figure: Impulse response of the exchange rate  $\Delta e_t$  to  $\psi_t$

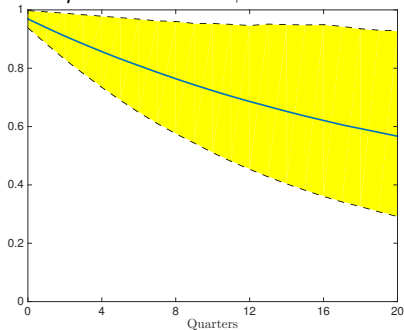
# Properties of the Exchange Rate

◀ back to slides

- Near-random-walk behavior (as  $\beta\rho \rightarrow 1$ )

$$\text{corr}(\Delta e_{t+1}, \Delta e_t) \rightarrow 0 \quad \frac{\text{var}(\Delta_k e_{t+k} - \mathbb{E}_t \Delta_k e_{t+k})}{\text{var}(\Delta_k e_{t+k})} \rightarrow 1 \quad \frac{\text{std}(\Delta e_t)}{\text{std}(\psi_t)} \rightarrow \infty$$

$\rho = 0.96$  and  $\beta = 0.99$



$\rho = 0.99$  and  $\beta = 0.995$

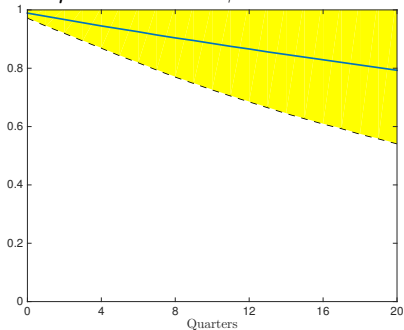


Figure: Contribution of the unexpected component (in small sample)

## RER Persistence

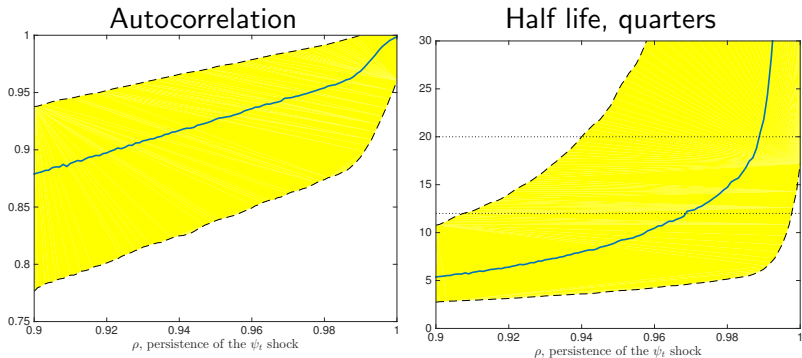
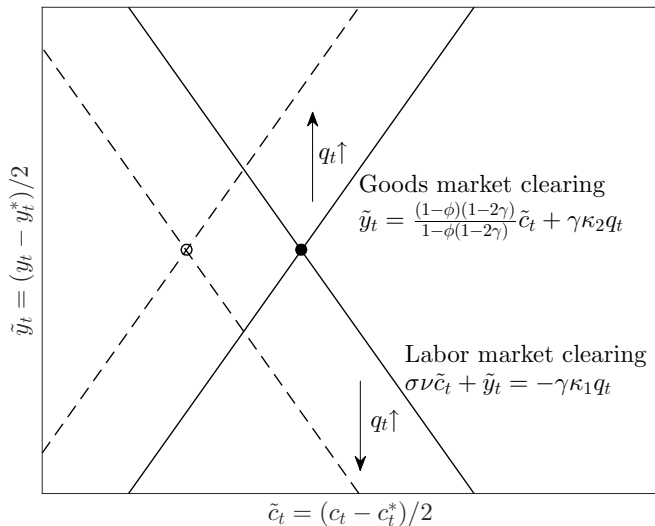


Figure: Persistence of the real exchange rate  $q_t$  in small samples

# Backus-Smith illustration

◀ back to slides



# Exchange Rate and Quantities

◀ back to slides

- Labor Supply:

$$\sigma \tilde{c}_t + \frac{1}{\nu} \tilde{l}_t = -\frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} q_t$$

— recall that:  $p_t = w_t + \frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} q_t$

- Labor Demand:

$$\tilde{l}_t = \tilde{y}_t + \frac{\phi}{1-\phi} \frac{\gamma}{1-2\gamma} q_t.$$

- Goods market clearing:

$$\tilde{y}_t = \frac{\zeta}{\zeta + \frac{2\gamma}{1-2\gamma}} \tilde{c}_t + \frac{2\theta(1-\alpha) \frac{1-\gamma}{1-2\gamma} - (1-\zeta)}{\zeta + \frac{2\gamma}{1-2\gamma}} \frac{\gamma}{1-2\gamma} q_t$$



# Exchange Rate and Interest Rate

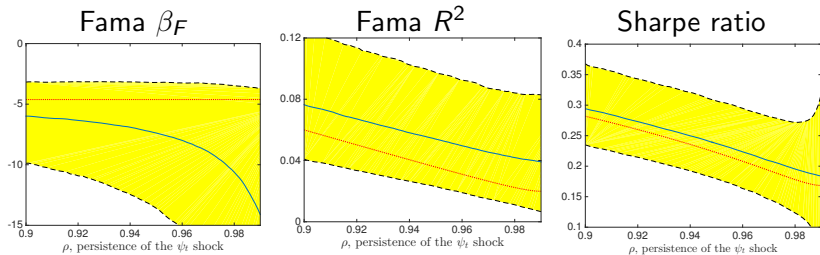


Figure: Deviations from UIP (in small samples)

# ER Disconnect: Robustness

◀ back to slides

	Data	Baseline	Robustness				
			$\theta = 2.5$	$\alpha = 0$	$\gamma = .15$	$\rho = 0.9$	$\sigma = 1$
1. $\rho(\Delta e)$	0.00	-0.02 (0.09)					-0.05
	$\rho(q)$	0.94	0.93* (0.04)				0.87
2. $HL(q)$	12.0	9.9* (6.4)					4.9
	$\sigma(\Delta q)/\sigma(\Delta e)$	0.98	0.75			0.54	
3.	$\sigma(\Delta s)/\sigma(\Delta q)$	0.34	0.30		1.16	0.46	
	$\sigma(\Delta q^P)/\sigma(\Delta q)$	0.98	1.10		1.16	1.26	
4.	$\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q)$	-0.25	-0.31	-0.42	-0.42	-0.81	-0.48
	Fama $\beta_F$	$\lesssim 0$	-8.1* (4.7)				
5.	Fama $R^2$	0.02	0.04 (0.02)				0.07
	$\sigma(i - i^*)/\sigma(\Delta e)$	0.06	0.03 (0.01)				
	Carry $SR$	0.20	0.21 (0.04)				0.29

Note: Baseline parameters:  $\gamma = 0.07$ ,  $\alpha = 0.4$ ,  $\theta = 1.5$ ,  $\rho = 0.97$ ,  $\sigma = 2$ ,  $\nu = 1$ ,  $\phi = 0.5$ ,  $\mu = 0$ ,  $\beta = 0.99$ . Results are robust to changing  $\nu$ ,  $\phi$ ,  $\mu$  and  $\beta$ . \*Asymptotic values:  $\rho(q) = 1$ ,  $HL(q) = \infty$ ,  $\beta_F = -4.6$ .

## Mechanism

- 1 An international asset demand shock  $\varepsilon_t > 0$  results in an immediate sharp ER depreciation to balance the asset market
- 2 Exchange rate then gradually appreciates (as the  $\psi_t$  shock wears out) to ensure the intertemporal budget constraint
- 3 Nominal and real devaluations happen together, and the real wage declines
- 4 Devaluation is associated with a dampened deterioration of the terms of trade and the resulting expenditure switching towards home goods
- 5 Consumption falls to ensure equilibrium in labor and goods markets
- 6 Consumption fall is supported by an increase in the interest rate, which balances out the fall in demand for domestic assets